GIC Option Valuation

A valuation model is presented for pricing a pool of GICs with an embedded redemption option. The model covers

- pricing of a closed (non-redeemable) GIC,
- penalty calculation for Redeemable and Premium GIC,
- embedded option valuation,
- numerical convergence of the embedded option pricing procedure.

We consider the following four GIC types:

• Non-Redeemable	A closed GIC, which periodically pays a fixed coupon and pays the principal at maturity.
• Redeemable	One month after inception till maturity (up to 7 years), the holder has an option to redeem the principal and accrued interest less a penalty based on the "call" rate specified by the exercise schedule; the schedule may include up to six contiguous windows with individual call rates.
• Prime	This GIC has three year maturity. During the two 30 day windows starting on inception anniversary dates, the holder has an option to redeem the principal and accrued interest less a penalty based on the call rates assigned to the respective windows.

- Flexible — A one year maturity GIC whose holder has an option to redeem the principal and accrued interest without any penalty from one month after inception till maturity. If the holder chooses to redeem the GIC within the first 30 days after inception, a zero call rate is applied.

We assume that the GIC holder receives deterministic payments at the specified payment dates.

Consider a GIC specified by

- maturity, T ,
- set of future payment times, $\{t_i\}_{i=1}^{N}$, where $t_1 < ... < t_N = T$.

Let

- c_{Cst} be the annualized customer coupon rate,
- c_{Tr} be the transfer coupon rate,
- f_c be the coupon rate compounding frequency,
- f_p be the coupon payment frequency (see table[1]),

We define an "equivalent simple annualized rate", which we denote EAR:

$$EAR_{Cst} = f_p \left[\left(1 + \frac{c_{Cst}}{f_c} \right)^{\frac{f_c}{f_p}} - 1 \right], \tag{1a}$$

$$EAR_{Tr} = f_p \left[\left(1 + \frac{c_{Tr}}{f_c} \right)^{\frac{f_c}{f_p}} - 1 \right] \tag{1b}$$

The payment P_i at time t_i is then

$$P_i = principal \times EAR_{Tr} \times (t_i - t_{i-1}), \qquad (2)$$

where the accrual period, $t_i - t_{i-1}$, is calculated using the ACT/365 day counting convention. We have previously reviewed the generation of the payment dates, t_i, and the respective payments, P_i,

Let $d_{Tr}(t)$ denote the price at valuation time of a zero coupon bond, based on the cost of funds rates, with maturity t and unit face value. The closed GIC transfer price, pv_{Tr}, is given by

$$pv_{Tr} = \sum_i d_{Tr}(t_i) P_i + d_{Tr}(t_N) \times principal, \qquad (3)$$

where the summation is over the remaining payment dates. To be specific, we first bootstrap a set of Cost of Funds (COF) discount factors $d_{Tr}(\tau_j)$ at the set of fixed times, $\{\tau_j = 0.5j\}_{j=1}^{M}$, where $\tau_1 < ... < \tau_M = T_1$. The discount factor $d_{Tr}(\tau)$, for $\tau_j < \tau < \tau_{j+1}$, where τ_j and τ_{j+1} are consecutive bootsrapping breakpoints, is given by the log-linear interpolation:

$$d_{Tr}(\tau) = \exp(-r(\tau)\tau),$$

where

$$r(\tau) = r(\tau_j) + \frac{(\tau - \tau_j)\lfloor r(\tau_{j+1}) - r(\tau_j) \rfloor}{\tau_{j+1} - \tau_j},$$

and

$$r(\tau_j) = -\frac{\log(d_{Tr}(\tau_j)))}{\tau_j}.$$

To compute the GIC value from the customer's perspective, pv_{cst}, we apply eq. (3) to customer discount factors. The customer discount factor bootstrapping algorithm is analogous to the COF discount factors bootstrapping

Assume that the GIC specified above can be redeemed at time *t* with the call rate r_{call}. We define an equivalent annualized simple call rate by

$$EAR_{call} = f_p\left[\left(1+\frac{r_{call}}{f_c}\right)^{\frac{f_c}{f_p}} - 1\right]. \qquad (4)$$

Let t_i, where $t_i \leq t$, be the coupon payment date that is immediately prior to time *t*. The time *t* redemption value is then

$$R_{Cst}(t) = principal \times \left[1 + (t - t_i)EAR_{Cst} - (EAR_{Cst} - EAR_{call})(t - t_0)\right] \qquad (5)$$

where t_0 is the GIC inception time. Note that the term

$$(EAR_{Cst} - EAR_{call})(t - t_0) \qquad (6)$$

in eq. (5) represents the penalty interest. For the customer, the intrinsic value of the embedded put (redemption) option is then

$$p_{Cst,\text{int}}(t) = \max\left(R_{Cst}(t) - pv_{Cst}(t), 0\right), \qquad (7)$$

where the closed GIC customer price pv_{Cst} is calculated as described in Section 3.1.

Let 1_t be an indicator process such that

$$1_t = \begin{cases} 1, & if\ t\ is\ in\ an\ exercise\ window, \\ 0, & otherwise. \end{cases}$$

Furthermore, let τ be the unique stopping time such that

$$E\left(\left(\frac{1_\tau\left(R_{Cst}(\tau) - pv_{Cst}(\tau)\right)}{e^{\int_t^\tau r_{Cst}(s)ds}}\right)^+ \middle| \Im_t\right) = \sup_{t<\tau<T} E\left(\left(\frac{1_\tau\left(R_{Cst}(\tau) - pv_{Cst}(\tau)\right)}{e^{\int_t^\tau r_{Cst}(s)ds}}\right)^+ \middle| \Im_t\right) \quad (7.a)$$

where

- the expectation is taken under the risk-neutral probability measure, and
- r_{Cst} denotes the customer short rate.

The put option's holding value is then

$$p_{Cst,hld}(t) = E\left(\left(\frac{1_\tau\left(R_{Cst}(\tau) - pv_{Cst}(\tau)\right)}{e^{\int_t^\tau r_{Cst}(s)ds}}\right)^+ \middle| \Im_t\right) \qquad (8)$$

The put option value at time t is

$$p_{Cst}(t) = \max\left(p_{Cst,\text{int}}(t), p_{Cst,hld}(t)\right), \tag{9}$$

where the choice of intrinsic value indicates customer's exercise at the current time.

Next, we define the following indicator process:

$$I_t = \begin{cases} 1, & \textit{if customer execercises at time } t, \\ 0, & \textit{otherwise}. \end{cases}$$

The intrinsic value of the redemption option is

$$p_{Tr,int}(t) = I_t\left(R_{Tr}(t) - pv_{Tr}(t)\right) \tag{10}$$

where

$$R_{Tr}(t) = principal \times \left[1 + (t - t_i)EAR_{Tr}\right] \tag{11}$$

We note that payoff function (10) is typically discontinuous.

The redemption option cost is then

$$E\left(\frac{p_{Tr,int}(\tau)}{e^{\int_0^\tau r_{Tr}(s)ds}} \middle| \Im_0\right), \tag{12}$$

where

- r_{Tr} denotes the short rate, and
- τ is the unique stopping time defined by eq. (7a)

We assume that the customer's short interest rates process satisfies a risk-neutral SDE of the Hull-White (HW) form,

$$dr_{cst} = (\theta_{cst}(t) - ar_{cst})dt + \sigma\, dW, \qquad (13)$$

where

- a is a constant mean reversion rate,
- σ is a constant volatility.

We assume that short interest rate, r_{Tr}, satisfies a similar risk-neutral SDE,

$$dr_{Tr} = (\theta_{Tr}(t) - ar_{Tr})dt + \sigma\, dW, \qquad (13.a)$$

where the a and σ parameters and standard Brownian motion W are the same as in

eq. (13). The drift term, $\theta(t)$, for each rate is calibrated to the respective initial interest rate term structure, which is bootsrapped as described in ref. [1]. For this purpose the algorithm requires as inputs

- HW short rate volatility and mean reversion parameter,
- basis yield curve key rates,
- key rate spreads (ref https://finpricing.com/lib/IrBasisCurve.html),
- customer key rate spreads.

We note that equations (13) and (13a) imply that the two short interest rates are perfectly correlated.